Пример. $\ln(x_1+\frac{1}{2})+\ln(x_2+8)+\ln(x_3+\frac{25}{2})+\ln(x_4+18)\to\max$

$A=22 \qquad x_1+x_2+x_3+x_4=22,\ x_i\geq 0,\ i=1,2,3,4$

$f_i(t)$ вогнутые, $f_1'(t)=\frac{1}{t+\frac{1}{2}}$, $f_2'(t)=\frac{1}{t+8}$, $f_3'(t)=\frac{1}{t+\frac{25}{2}}$,

$f_4'(t)=\frac{1}{t+18}$, $f_1'(0)=2$, $f_2'(0)=\frac{1}{8}$, $f_3'(0)=\frac{2}{25}$, $f_4'(0)=\frac{1}{18}$

решаем систему $k=4$

$\frac{1}{x_1+\frac{1}{2}}=C$, $\frac{1}{x_2+8}=C$, $\frac{1}{x_3+\frac{25}{2}}=C$, $\frac{1}{x_4+18}=C \Rightarrow$

$\frac{1}{C}=x_1+\frac{1}{2}$, $\frac{1}{C}=x_2+8$, $\frac{1}{C}=x_3+\frac{25}{2}$ $\|+\|$ $\frac{1}{C}=x_4+18 \Rightarrow \frac{4}{C}=22+39=61$

$x_4=\frac{61}{4}-18<0 \Rightarrow x_4=0$, $k=3$, $\frac{3}{C}=22+21=43 \Rightarrow \frac{1}{C}=\frac{43}{3}$

$C=\frac{3}{43}>\frac{1}{18}=f_4'(0)$, $\begin{cases} x_1^0=\frac{43}{3}-\frac{1}{2}=\frac{83}{6}, & x_2^0=\frac{43}{3}-8=\frac{19}{3}, \quad x_3^0=\frac{43}{3}-\frac{25}{2}=\frac{11}{6} \\ x_4^0=0 \end{cases}$

Дискретная задача.

$x^{(1)} = (14, 6, 2, 0)$ Проверка условия оптимальности

$\ln(x-1) - \ln x = \ln\left(\frac{x-1}{x}\right)$

| $i$ | $x_i^{(1)}$ | $f_i(x_i^{(1)}+1) - f_i(x_i^{(1)})$ | $f_i(x_i^{(1)}) - f_i(x_i^{(1)}-1)$ |
|---|---|---|---|
| 1 | 14 | $\ln(31/29)$ | $\ln(29/27)$ |
| 2 | 6 | $\ln(15/14)$ | $\ln(14/13)$ |
| 3 | 2 | $\ln(31/29)$ | $\ln(29/27)$ |
| 4 | 0 | — | |

$\frac{14}{13} > \frac{15}{14}$

$169 > 165$

$\Rightarrow x^* = (14, 6, 2, 0)$